5.3 Exercise Set

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Concept Reinforcement *Classify each of the following statements as either true or false.*

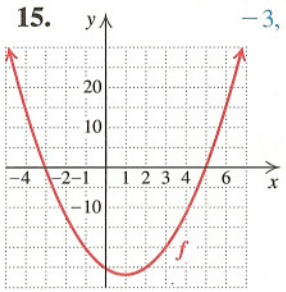
1. The largest common factor of $10x^4 + 15x^2$ is $5x$. False
2. The largest common factor of a polynomial always has the same degree as the polynomial itself. False
3. The polynomial $8x + 9y$ is prime. True
4. When the leading coefficient of a polynomial is negative, we generally factor out a common factor with a negative coefficient. True
5. A polynomial is not prime if it contains a common factor other than 1 or -1. True
6. All polynomials with four terms can be factored by grouping. False
7. The expressions $b - a$, $-(a - b)$, and $-1(a - b)$ are all equivalent. True
8. The complete factorization of $12x^3 - 20x^2$ is $4x(3x^2 - 5x)$. False

Tell whether each of the following is an expression or an equation.

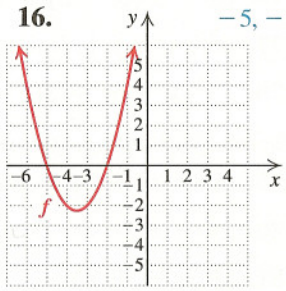
9. $x^2 + 6x + 9$ Expression
10. $x^3 = x^2 - x + 3$ Equation
11. $3x^2 = 3x$ Equation
12. $x^4 + 3x^3 + x^2$ Expression
13. $2x^3 + x^2 = 0$ Equation
14. $5x^4 + 5x$ Expression

In Exercises 15 and 16, use the graph to solve $f(x) = 0$.

15. $-3, 5$

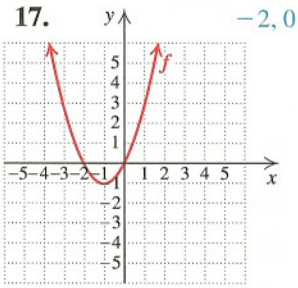


16. $-5, -2$

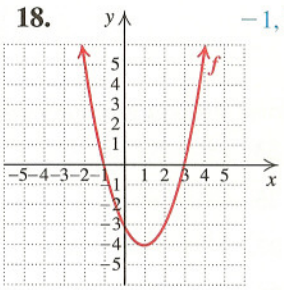


In Exercises 17 and 18, use the graph to find the zeros of the function f.

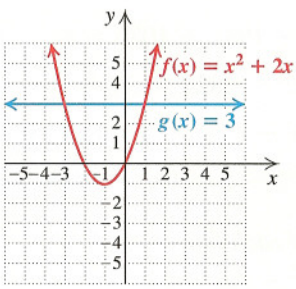
17. $-2, 0$



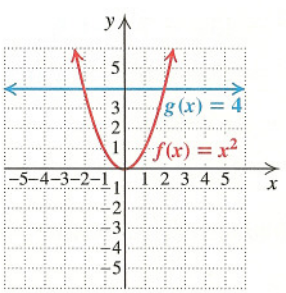
18. $-1, 3$



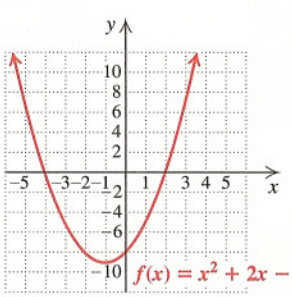
19. Use the graph below to solve $x^2 + 2x = 3$. $-3, 1$



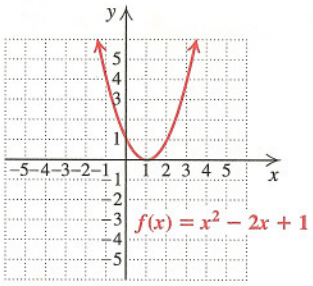
20. Use the graph below to solve $x^2 = 4$. $-2, 2$



21. Use the graph below to solve $x^2 + 2x - 8 = 0$. $-4, 2$



22. Use the graph below to find the zeros of the function given by $f(x) = x^2 - 2x + 1$. 1



Solve using a graphing calculator.

23. $x^2 = 5x$ 0, 5

24. $2x^2 = 20x$ 0, 10

25. $4x = x^2 + 3$ 1, 3

26. $x^2 = 1$ −1, 1

27. $x^2 + 150 = 25x$ 10, 15

28. $2x^2 + 25 = 51x$ 0.5, 25

29. $x^3 - 3x^2 + 2x = 0$ 0, 1, 2

30. $x^3 + 2x^2 = x + 2$ −2, −1, 1

31. $x^3 - 3x^2 - 198x + 1080 = 0$ −15, 6, 12

32. $2x^3 + 25x^2 - 282x + 360 = 0$ −20, 1.5, 6

33. $21x^2 + 2x - 3 = 0$ −0.42857, 0.33333

34. $66x^2 - 49x - 5 = 0$ −0.09091, 0.83333

Find the zeros of each function.

35. $f(x) = x^2 - 4x - 45$ −5, 9

36. $g(x) = x^2 + x - 20$ −5, 4

37. $p(x) = 2x^2 - 13x - 7$ −0.5, 7

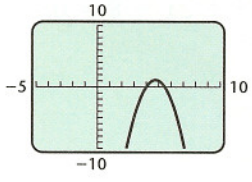
38. $f(x) = 6x^2 + 17x + 6$ −2.42013, −0.41320

39. $f(x) = x^3 - 2x^2 - 3x$ −1, 0, 3

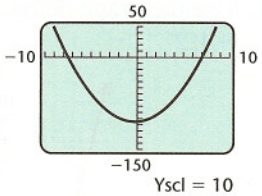
40. $r(x) = 3x^3 - 12x$ −2, 0, 2

Aha! *Match each graph to the corresponding function in Exercises 41–44.*

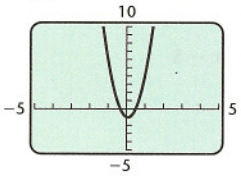
I



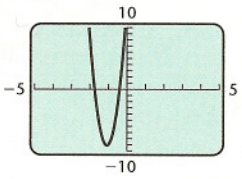
II



III



IV



41. $f(x) = (2x - 1)(3x + 1)$ III

42. $f(x) = (2x + 15)(x - 7)$ II

43. $f(x) = (4 - x)(2x - 11)$ I

44. $f(x) = (5x + 2)(4x + 7)$ IV

Write an equivalent expression by factoring out the greatest common factor.

45. $2t^2 + 8t$ $2t(t + 4)$

46. $3y^2 - 6y$ $3y(y - 2)$

47. $9y^3 - y^2$ $y^2(9y - 1)$

48. $x^3 + 8x^2$ $x^2(x + 8)$

49. $15x^2 - 5x^4 + 5x$ $5x(3x - x^3 + 1)$

50. $8y^2 + 4y^4 - 2y$ $2y(4y + 2y^3 - 1)$

51. $4x^2y - 12xy^2$ $4xy(x - 3y)$

52. $5x^2y^3 + 15x^3y^2$ $5x^2y^2(y + 3x)$

53. $3y^2 - 3y - 9$ $3(y^2 - y - 3)$

54. $15x^2 - 5x + 5$ $5(3x^2 - x + 1)$

55. $6ab - 4ad + 12ac$ $2a(3b - 2d + 6c)$

56. $8xy + 10xz - 14xw$ $2x(4y + 5z - 7w)$

57. $72x^3 - 36x^2 + 24x$ $12x(6x^2 - 3x + 2)$

58. $12a^4 - 21a^3 - 9a^2$ $3a^2(4a^2 - 7a - 3)$

59. $x^5y^5 + x^4y^3 + x^3y^3 - xy^2$ $xy^2(x^4y^3 + x^3y + x^2y - 1)$

60. $x^9y^6 - x^7y^5 + x^4y^4 + x^3y$ $x^3y(x^6y^5 - x^4y^4 + xy^3 + 1)$

61. $9x^3y^6z^2 - 12x^4y^4z^4 + 15x^2y^5z^3$ ⊡

62. $14a^4b^3c^5 + 21a^3b^5c^4 - 35a^4b^4c^3$ $7a^3b^3c^3(2ac^2 + 3b^2c - 5ab)$

Write an equivalent expression by factoring out a factor with a negative coefficient.

63. $-5x + 35$ $-5(x - 7)$

64. $-6y - 72$ $-6(y + 12)$

65. $-2x^2 + 4x - 12$ $-2(x^2 - 2x + 6)$

66. $-2x^2 + 12x + 40$ $-2(x^2 - 6x - 20)$

67. $3y - 24x$ $-3(-y + 8x)$, or $-3(8x - y)$

68. $7x - 56y$ ⊡

69. $-x^2 + 5x - 9$ $-(x^2 - 5x + 9)$

70. $-p^3 - 4p^2 + 11$ $-(p^3 + 4p^2 - 11)$

71. $-a^4 + 2a^3 - 13a$ $-a(a^3 - 2a^2 + 13)$

72. $-m^{10} - m^9 + m^8 - 2m^7$ $-m^7(m^3 + m^2 - m + 2)$

Write an equivalent expression by factoring.

73. $a(b - 5) + c(b - 5)$ $(b - 5)(a + c)$

74. $r(t - 3) - s(t - 3)$ $(t - 3)(r - s)$

75. $(x + 7)(x - 1) + (x + 7)(x - 2)$ ⊡

76. $(a + 5)(a - 2) + (a + 5)(a + 1)$ ⊡

77. $a^2(x - y) + 5(y - x)$ $(x - y)(a^2 - 5)$

78. $5x^2(x - 6) + 2(6 - x)$ $(x - 6)(5x^2 - 2)$

Factor by grouping, if possible, and check.

79. $ac + ad + bc + bd$ $(c + d)(a + b)$

80. $xy + xz + wy + wz$ $(y + z)(x + w)$

81. $b^3 - b^2 + 2b - 2$ $(b - 1)(b^2 + 2)$

82. $y^3 - y^2 + 3y - 3$ $(y - 1)(y^2 + 3)$

83. $x^3 - x^2 - 2x + 5$ Not factorable by grouping

⊡ Answers to Exercises 61, 68, 75, and 76 are on p. IA-12.

84. $p^3 + p^2 - 3p + 10$ Not factorable by grouping

85. $a^3 - 3a^2 + 6 - 2a$ $(a - 3)(a^2 - 2)$

86. $t^3 + 6t^2 - 2t - 12$ $(t + 6)(t^2 - 2)$

87. $x^6 - x^5 - x^3 + x^4$ $x^3(x - 1)(x^2 + 1)$

88. $y^4 - y^3 - y + y^2$ $y(y - 1)(y^2 + 1)$

89. $2y^4 + 6y^2 + 5y^2 + 15$ $(y^2 + 3)(2y^2 + 5)$

90. $2xy - x^2y - 6 + 3x$ $(2 - x)(xy - 3)$

91. *Height of a Baseball.* A baseball is popped up with an upward velocity of 72 ft/sec. Its height in feet, $h(t)$, after t seconds is given by

$$h(t) = -16t^2 + 72t.$$

a) Find an equivalent expression for $h(t)$ by factoring out a common factor with a negative coefficient. $h(t) = -8t(2t - 9)$

b) Perform a partial check of part (a) by evaluating both expressions for $h(t)$ at $t = 1$. $h(1) = 56$ ft

92. *Height of a Rocket.* A water rocket is launched upward with an initial velocity of 96 ft/sec. Its height in feet, $h(t)$, after t seconds is given by

$$h(t) = -16t^2 + 96t.$$

a) Find an equivalent expression for $h(t)$ by factoring out a common factor with a negative coefficient. $h(t) = -16t(t - 6)$

b) Check your factoring by evaluating both expressions for $h(t)$ at $t = 1$. $h(1) = 80$ ft

93. *Airline Routes.* When an airline links n cities so that from any one city it is possible to fly directly to each of the other cities, the total number of direct routes is given by

$$R(n) = n^2 - n.$$

Find an equivalent expression for $R(n)$ by factoring out a common factor. $R(n) = n(n - 1)$

94. *Surface Area of a Silo.* A silo is a structure that is shaped like a right circular cylinder with a half sphere on top. The surface area of a silo of height h and radius r (including the area of the base) is given by the polynomial $2\pi rh + \pi r^2$. (Note that h is the height of the entire silo.) Find an equivalent expression by factoring out a common factor. $\pi r(2h + r)$

95. *Total Profit.* When x hundred gaming systems are sold, Rolics Electronics collects a profit of $P(x)$, where

$$P(x) = x^2 - 3x, \quad P(x) = x(x - 3)$$

and $P(x)$ is in thousands of dollars. Find an equivalent expression by factoring out a common factor.

96. *Total Profit.* After t weeks of production, Claw Foot, Inc., is making a profit of $P(t) = t^2 - 5t$ from sales of their surfboards. Find an equivalent expression by factoring out a common factor. $P(t) = t(t - 5)$

97. *Total Revenue.* Urban Sounds is marketing a new MP3 player. The firm determines that when it sells x units, the total revenue R is given by the polynomial function

$$R(x) = 280x - 0.4x^2 \text{ dollars.}$$

Find an equivalent expression for $R(x)$ by factoring out $0.4x$. $R(x) = 0.4x(700 - x)$

98. *Total Cost.* Urban Sounds determines that the total cost C of producing x MP3 players is given by the polynomial function

$$C(x) = 0.18x + 0.6x^2.$$

Find an equivalent expression for $C(x)$ by factoring out $0.6x$. $C(x) = 0.6x(0.3 + x)$

99. *Counting Spheres in a Pile.* The number N of spheres in a triangular pile like the one shown here is a polynomial function given by

$$N(x) = \tfrac{1}{6}x^3 + \tfrac{1}{2}x^2 + \tfrac{1}{3}x,$$

where x is the number of layers and $N(x)$ is the number of spheres. Find an equivalent expression for $N(x)$ by factoring out $\frac{1}{6}$. $N(x) = \frac{1}{6}(x^3 + 3x^2 + 2x)$

$f(n) = \frac{1}{2}(n^2 - n)$

100. *Number of Games in a League.* If there are n teams in a league and each team plays every other team once, we can find the total number of games played by using the polynomial function $f(n) = \frac{1}{2}n^2 - \frac{1}{2}n$. Find an equivalent expression by factoring out $\frac{1}{2}$.

101. *High-fives.* When a team of n players all give each other high-fives, a total of $H(n)$ hand slaps occurs, where

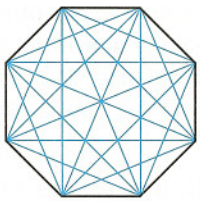
$$H(n) = \tfrac{1}{2}n^2 - \tfrac{1}{2}n. \quad H(n) = \tfrac{1}{2}n(n - 1)$$

Find an equivalent expression by factoring out $\frac{1}{2}n$.

102. *Number of Diagonals.* The number of diagonals of a polygon having n sides is given by the polynomial function

$$P(n) = \tfrac{1}{2}n^2 - \tfrac{3}{2}n.$$

Find an equivalent expression for $P(n)$ by factoring out $\frac{1}{2}n$. $P(n) = \frac{1}{2}n(n-3)$



Solve using the principle of zero products.

103. $(x+3)(x-4) = 0$ $-3, 4$

104. $(x+10)(x+11) = 0$ $-11, -10$

105. $x(x+1) = 0$ $-1, 0$

106. $5x(x-2) = 0$ $0, 2$

107. $x^2 - 3x = 0$ $0, 3$

108. $2x^2 + 8x = 0$ $-4, 0$

109. $-5x^2 = 15x$ $-3, 0$

110. $2x - 4x^2 = 0$ $0, \frac{1}{2}$

111. $12x^4 + 4x^3 = 0$ $-\frac{1}{3}, 0$

112. $21x^3 = 7x^2$ $0, \frac{1}{3}$

113. Given that $f(x) = (x-3)(x+7)$, find all values of a for which $f(a) = 0$. $-7, 3$

114. Given that $f(x) = (3x+1)(x+8)$, find all values of a for which $f(a) = 0$. $-8, -\frac{1}{3}$

115. Given that $f(x) = 2x(5x+9)$, find all values of a for which $f(a) = 0$. $0, -\frac{9}{5}$

116. Given that $f(x) = 8x(x-1)$, find all values of a for which $f(a) = 0$. $0, 1$

117. Given that $f(x) = x^3 - 3x^2$, find all values of a for which $f(a) = 0$. $0, 3$

118. Given that $f(x) = 6x + 9x^2$, find all values of a for which $f(a) = 0$. $0, -\frac{2}{3}$

TW **119.** Write a two-sentence paragraph in which the word "factor" is used at least once as a noun and once as a verb.

TW **120.** Jasmine claims that the zeros of the function given by $f(x) = x^4 - 3x^2 + 7x + 20$ are $-1, 1, 2, 4$, and 5. How can you tell, without performing any calculations, that she cannot be correct?

SKILL REVIEW

To prepare for Section 5.4, review multiplying binomials using FOIL (Section 5.2).

Multiply. [5.2]

121. $(x+2)(x+7)$ $x^2 + 9x + 14$

122. $(x-2)(x-7)$ $x^2 - 9x + 14$

123. $(x+2)(x-7)$ $x^2 - 5x - 14$

124. $(x-2)(x+7)$ $x^2 + 5x - 14$

125. $(a-1)(a-3)$ $a^2 - 4a + 3$

126. $(t+3)(t+5)$ $t^2 + 8t + 15$

127. $(t-5)(t+10)$ $t^2 + 5t - 50$

128. $(a+4)(a-6)$ $a^2 - 2a - 24$

SYNTHESIS

TW **129.** Ashlee factors $8x^2y - 10xy^2$ as

$$2xy \cdot 4x - 2xy \cdot 5y.$$

Is this the factorization of the polynomial? Why or why not?

TW **130.** What is wrong with solving $x^2 = 3x$ by dividing both sides of the equation by x?

131. Use the results of Exercise 21 to factor $x^2 + 2x - 8$. $(x+4)(x-2)$

132. Use the results of Exercise 22 to factor $x^2 - 2x + 1$. $(x-1)(x-1)$ $x^5y^4 + x^4y^6 = x^4y^4(x+y^2)$

Complete each of the following factorizations.

133. $x^5y^4 + \_\_\_ = x^4y^4(\_\_\_ + y^2)$

134. $a^3b^7 - \_\_\_ = \_\_\_(ab^4 - c^2)$ $a^3b^7 - a^2b^3c^2 = a^2b^3(ab^4 - c^2)$

Write an equivalent expression by factoring out the smallest power of x in each of the following.

135. $x^{-6} + x^{-9} + x^{-3}$ $x^{-9}(x^3 + 1 + x^6)$

136. $x^{-8} + x^{-4} + x^{-6}$ $x^{-8}(1 + x^4 + x^2)$

137. $x^{1/3} - 5x^{1/2} + 3x^{3/4}$ $x^{1/3}(1 - 5x^{1/6} + 3x^{5/12})$

138. $x^{3/4} + x^{1/2} - x^{1/4}$ $x^{1/4}(x^{1/2} + x^{1/4} - 1)$

Factor.

Aha! **139.** $5x^5 - 5x^4 + x^3 - x^2 + 3x - 3$ $(x-1)(5x^4 + x^2 + 3)$

140. $ax^2 + 2ax + 3a + x^2 + 2x + 3$ $(x^2 + 2x + 3)(a+1)$

Write an equivalent expression by factoring. Assume that all exponents are natural numbers.

141. $2x^{3a} + 8x^a + 4x^{2a}$ $2x^a(x^{2a} + 4 + 2x^a)$

142. $3a^{n+1} + 6a^n - 15a^{n+2}$ $3a^n(a + 2 - 5a^2)$